Imaginary Numbers

The universe of numbers keeps expanding:

Whole numbers: 1, 2, 3, 4, 5, 6, ...

- addition & multiplication work great
- ________ can be problematic ...

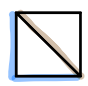
Introduce ________ numbers and ____:

... −6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, ...

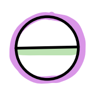
- added bonus: use zero as ________, efficiently keep track of larger & larger numbers, addition in columns, multiplication...
- ______ can be problematic

Introduce ______ & ______.

- may wonder about decimal expansions that never end & never repeat: do they ____ anything (need notion of ____)
- geometric questions
  - side length & ______ of square
  - diameter & ________ of circle

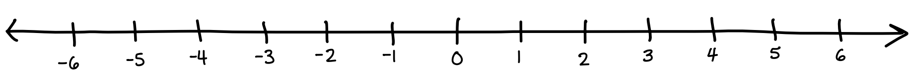


$\frac{\text{diag.}}{\text{side}} = \boxed{}$



$\frac{\text{circ}}{\text{diam.}} = \boxed{}$

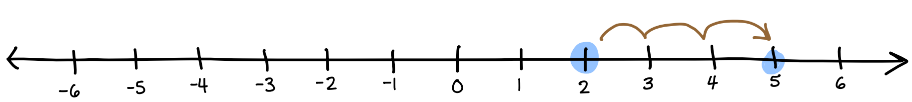
Introduce irrational numbers. Now have complete number line.



Arithmetic on the number line.

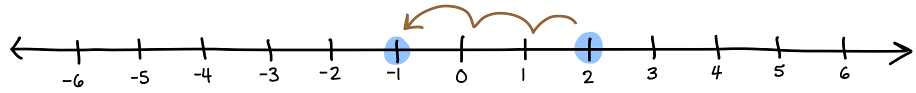
- Addition: ______ right or left.

  2+3: Start at 2 & shift 3 units right



2+3=5

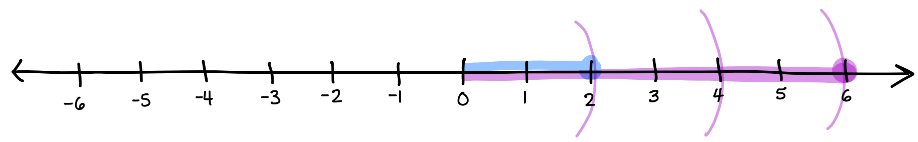
$2+(-3)$: Start at 2 & shift 3 units left.



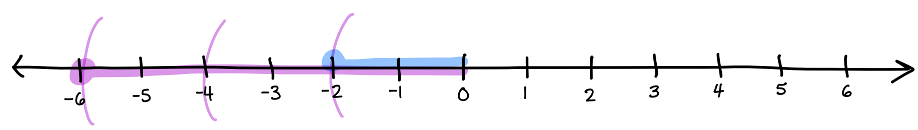
$2-3=-1$

- Multiplication by a positive number : ______ (or ______)

  2×3 : Start on right, stretch 3 times farther to right.

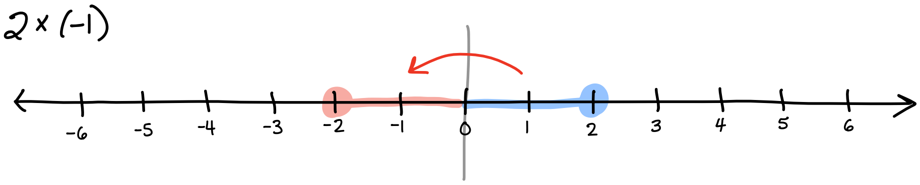


  -2×3 : Start on left, stretch 3 times farther to left.

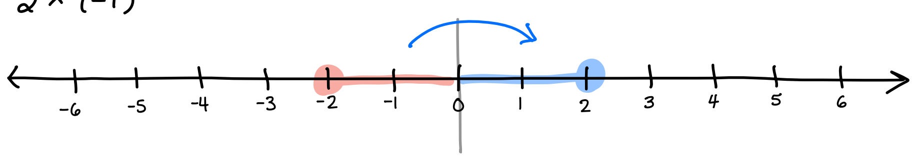


- Multiplication by -1 : ______

  $2\times(-1)$



  $-2\times(-1)$



- Multiplication by negative : flip & stretch (or shrink)

  ↳ Negative times negative is ______.

  (Start left, flip & end up on ______)

Notice : For any number x,

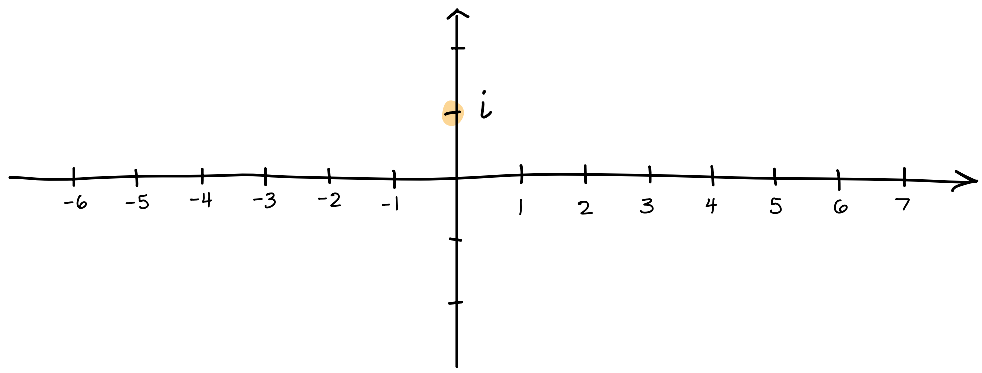
- if x is positive, x^2 is ______
- if x is negative, x^2 is ______
- if x is zero, x^2 is ______

} x^2 is not ______ in any case

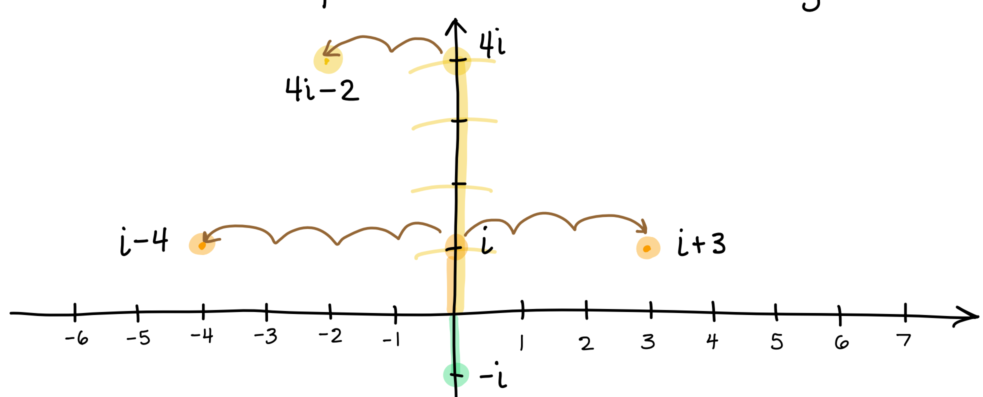
So if there is a number x with $x^2 = -1$, then x is not ______, not ______, and not ______... not on the ______!

Enlarging the universe of numbers to include "i", with $i^2 = -1$.

- Cannot live on the number line.
- Add an extra dimension to accommodate it!



Once we include i, we may include more numbers, obtained by addition & multiplication... ______ ("hybrid") numbers



Multiplication by -1 flips i ______.

Can also think of it as ______ rotation.

Plot the complex numbers :

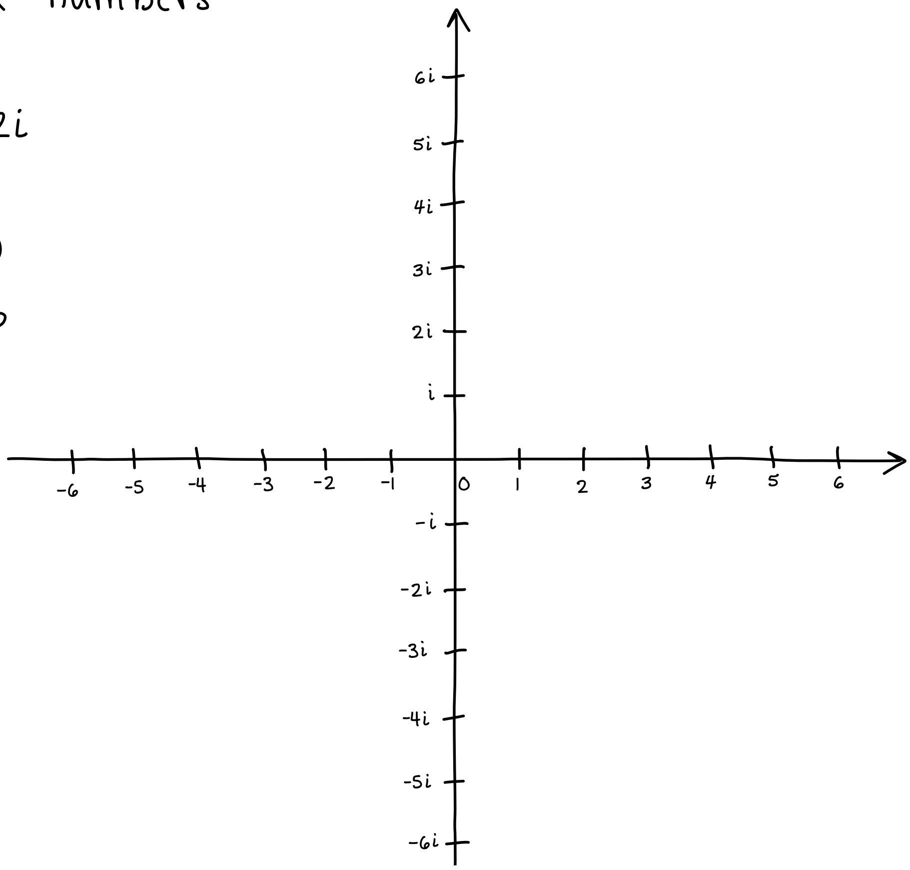
A. $2 + 3i$

B. $(2+3i) + 2i$

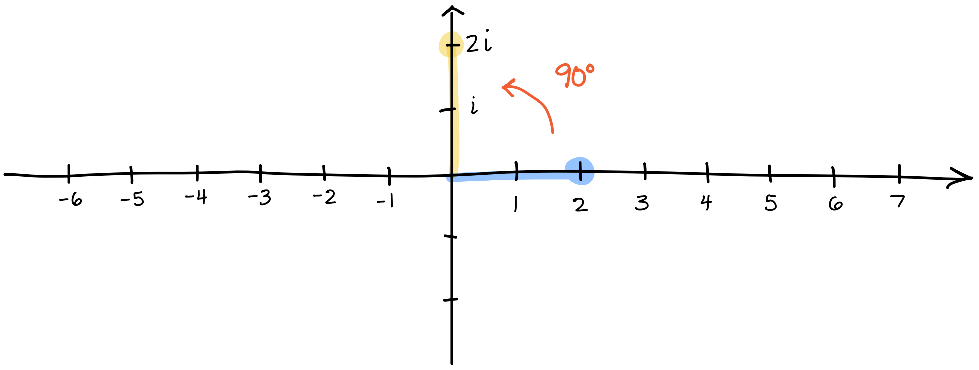
C. $-(2+3i)$

D. $-2(2+3i)$

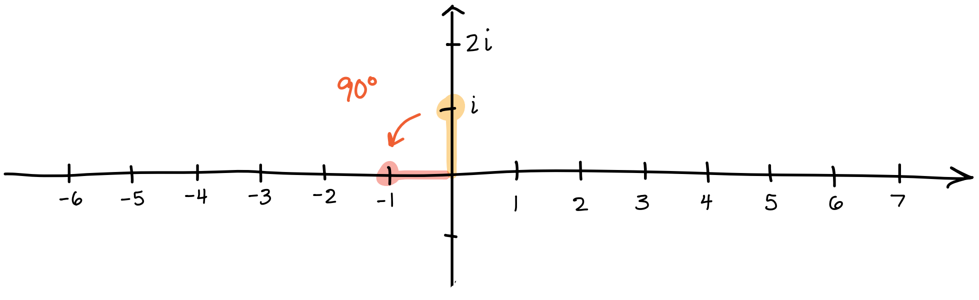
E. $(2+3i) - 6$



Multiplication by i : $90°$ counterclockwise rotation



So $i^2 = i \times i$ is a $90°$ rotation from "North" ... $i^2 =$ ____



Plot the complex numbers :

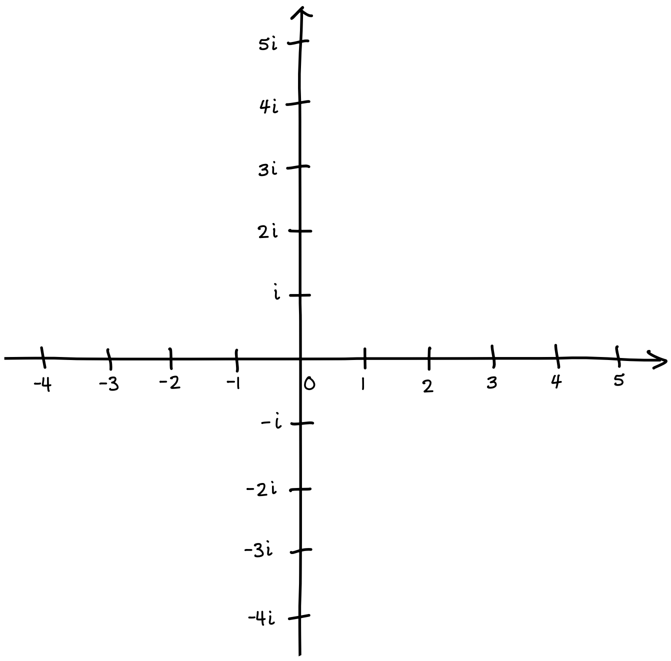
A. $4+3i$

B. $(4+3i)\cdot i$

C. $(-3+4i)\cdot i$

D. $(-4-3i)\cdot i$

E. $(3-4i)\cdot i$



Okay ... but why?

- Historically : The search for _____ of "_____________."

$$x^3-6x+11x-6=(x-1)(x-2)(x-3)$$

↑ How to figure out?

Now just as "real" & integrated into modern math as any other Kind of number.

- Practical Applications :
  - electromagnetic waves, vibrations on bridges, ...
  - Dirac, the positron, & PET scans

- Beauty! Newton's Method → Complex Dynamics (Hubbard)

  Euler's Formula :

- Another benefit of doing math : stretches the __________.